SUBSTRATE REGULATION IN FIXED BED BIOREACTORS VIA FEEDBACK CONTROL

REGULACION DEL SUBSTRATO DE SALIDA EN BIOREACTORES DE LECHO FIJO VIA CONTROL RETROALIMENTADO

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Abstract

The main issue of this work is the synthesis of robust nonlinear PID control law for the regulation of the outlet substrate concentration in a distributed parameter system, a fixed bed bioreactor. The controller design is realised at outlet reactor conditions via a linearising control law, which is coupled with a proportional-derivative reduced order observer. Therefore, it is possible to infer on-line the unknown kinetic terms. Controller structure is analogous to PID structure, where novel tuning rules are given in terms of the system structure and estimation and closed-loop time constants. This control strategy is robust against noisy substrate concentration measurements, sustained disturbances and model uncertainties. Performance of the outlet substrate concentration in the fixed bed bioreactor is compared, by means of numerical simulations, when the proposed control scheme and standard PID controllers are used.

Keywords: distributed parameter systems, fixed bed bioreactor, momentum balance, robust regulation of concentration, PID controller, tuning rules.

Resumen

El objetivo principal de este trabajo es la síntesis de una ley de control no lineal robusto tipo PID para la regulación de la concentración de substrato en un sistema de parámetros distribuidos, un bioreactor de lecho fijo. El diseño del control se realiza a las condiciones de salida del reactor vía una ley de control linealizante, la cual se acopla a un observador proporcional-derivativo de orden reducido. Así se logra inferir en línea la cinética desconocida del proceso. La estructura del control resultante es análoga a un PID, donde se dan nuevas reglas de sintonizado en términos de la estructura del sistema y las constantes de tiempo de estimación y operación a lazo cerrado. Esta estrategia de control es robusta contra mediciones ruidosas de la concentración de substrato, perturbaciones sostenidas e incertidumbres en el modelo. El desempeño de la concentración de substrato a la salida del bioreactor se compara, por medio de simulaciones numéricas, utilizando el control propuesto y controles PID estándar.

Palabras clave: balance de momentum, bioreactor de lecho fijo, control PID, reglas de sintonizado, regulación robusta de la concentración, sistemas de parámetros distribuidos.

1. Introduction

Continuous tubular reactors are of great importance in biochemical industry because of their high yields, large flow velocities, and efficient product extraction. These reactors are used, for example, in biofilters, ethanol production, anaerobic digestion and wastewater treatment (Hodge and Devinny, 1995; Bastin and Dochain, 1990). This kind of bioreactors is modelled as distributed parameter systems; therefore, a set of Partial Differential Equations is obtained. However, control theory for these systems is still under development.

Two different methodologies have been proposed for the control of distributed parameter systems (Christofedes and Daoutidis, 1998). In the first one, the control algorithm is based on a distributed parameter system, which is reduced to a finite dimensional model.

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However, model uncertainties make this approach inadequate in practice. In the second approach, the distributed parameter system is reduced to a set of ordinary differential equations (Wallman and Foss, 1981). This reduced order model keeps the essential dynamics of the process and is useful for regulation purposes. Moreover, analyses in terms of stability and performance are easily done. In addition, the problem is tractable because the system is represented by a finite dimensional set of ordinary differential equations. Therefore, most control research has followed this second approach.

About the control of fixed bed reactors, the use of input-output linearising control techniques have been proposed. These approaches, however, need precise knowledge of the nonlinear dynamics of the system; therefore, realisation problems arise during application of these methodologies (Slotine and Li, 1991). Also sliding control techniques (Hanczyc and Palazoglu, 1995) have been studied with good performance, but the implementation for plant operators of these advanced control strategies is not easy enough. For these reason the standard Proportional-Integral-Derivative (PID) controller is generally employed in commercial hardware for industrial plants.

Other studies on robust control have proposed the tuning of PID controllers by using information related to the model structure, such as the Internal Model Control (IMC). These tuning rules seem to be more robust than semi-empirical rules, for example those proposed by Ziegler-Nichols and Cohen-Coon. One advantage of this technique is that the physical interpretation of control parameters is closed to model parameters. Therefore, proper tuning of a PID has the same effect as the use of another more efficient controllers (Morari and Zafirou, 1989).

Robust control of tubular reactors has been previously studied (Aguilar *et al*., 2002a, 2002b) in the case of fast catalyst deactivation. In this kind of reactors both phases, solid and fluid, move together. The control methodology applied was successful for regulation and servocontrol. In this paper, that methodology is extended to a class of fixed bed bioreactor, which contains immobilised biomass. In this process, the solid phase is fixed and the fluid phase is moving. This process presents a number of interesting features such as the close correlation of reactor yields and composition to flow velocities and the catalytic activity that decays as function of the composition of the substrate. Although operational experience with this class of bioreactors has been satisfactory, the lack of the possibility of direct control of yields and compositions is a matter of concern. Moreover, some of the feedstock properties are difficult to estimate, especially the reaction rate. The controlled variable is the outlet substrate concentration and the manipulated variable is the input axial velocity related to volumetric flow (Dochain *et al*., 1992).

A finite dimensional model is generated by spatial discretisation of a distributed parameter model for a fixed bed bioreactor. Then, an input-output linearising controller based on uncertainty estimation is proposed for the control of the outlet substrate concentration. The uncertainty (reaction rate) estimator consists of proportional and derivative contributions of the estimation error. The derivative action of the estimator generates fast estimates of the reaction rate; helping to deal with lag in the estimation and control algorithm produced by the distributed parameter system dynamics. When the estimator is coupled to the proposed control law, PID controller structure is obtained. The controller tuning procedure is reduced to a selection of the proportional, integral and derivative gains, following the proposed tuning rules in a straightforward manner.

2. Model for the fixed bed bioreactor

The mathematical model for fixed bed bioreactors are generated by the classical application of mass balances for biomass, and substrate concentrations and momentum balance for axial velocity (axially there is not mixing). These conservation equations give a set of first order hyperbolic partial differential equations, which must be considered assuming that biomass (microorganisms playing the role of self-catalysts) is fixed in the bed of the bioreactor. There are other biochemical compounds, related to the generated products and the substrate, which flow through the bioreactor. The bioreactor cross-section is constant and the diffusion transport is negligible with respect to the convective transport.

Under these assumptions, the mathematical model of the bioreactor has the same structure as that in Bastin and Dochain (1990). In order to take into account delayed flows due to convective transport into the fixed bed, in this work momentum balance is introduced. This produces lags in control actions. In fact, momentum balance takes more importance when the flow under study is compressible; in that case, velocity propagation waves may be present along the bed.

The equations for the biomass concentration, axial velocity and substrate concentration are, respectively:

$$\frac{\partial x_1}{\partial t} = \mu(x_3)x_1 \tag{1}$$

$$\frac{\partial x_2}{\partial t} + X_2 \frac{\partial x_2}{\partial Z} = 0 \tag{2}$$

$$\frac{\partial x_3}{\partial t} + \frac{\partial (x_2 x_3)}{\partial Z} = -\mathrm{Y}_1 \mu(x_3) x_1 \tag{3}$$

Here x_1 and x_3 denote biomass and substrate concentrations, respectively, x_2 is the axial fluid velocity, $\mu(x_3)$ is the specific growth rate, Y_1 is the substrate / biomass yield coefficient and Z is the axial coordinate.

As it was mentioned above, there are several methodologies to approximate distributed parameter mathematical models. In this paper, a first-order forward difference approximation for the temporal derivatives and a first-order backward difference approximation for the spatial derivative were chosen. This procedure yields a reduced order model represented by ordinary differential equations. It is important to note that the use of finite difference discretisation preserves the mathematical structure of the balances inside the bioreactor.

A straightforward application of first backward explicit difference approximation $\left[(\Delta t)^2 \quad (\Delta Z)\right]$ of the derivatives along the tube axis yields the following finite-dimensional model:

$$\frac{\partial x_{1,i}}{\partial t} = \mu_i x_{1,i} \quad for\ i = 1,2,...,n \tag{4}$$

$$\frac{\partial x_{2,i}}{\partial t} = -x_{2,i} \frac{x_{2,i} - x_{2,i-1}}{\xi} \quad for \quad i = 1,2,...,n \tag{5}$$

$$\frac{\partial x_{3,i}}{\partial t} = -x_{2,i} \frac{x_{3,i} - x_{3,i-1}}{\xi} - x_{3,i} \frac{x_{2,i} - x_{2,i-1}}{\xi} - \mathrm{Y}_{1i} \mu_i x_{1,i} \quad for \quad i = 1,2,...,n \tag{6}$$

Here i denotes the discretisation point, n is the number of inner discretisation points and ξ is the constant length between each inner discretisation point. In this way, the original *partial* differential model- Eq. 1 to 3 are is reduced to a set of 3*n* *ordinary* differential equations. A basic discrete perturbation stability analysis of the discretised Eqs. 4 to 6 provided insight into the limitations on step sizes that are needed to obtain a stable solution. In this case, the Burgers structure of the Eq. 5 imposes the stability requirements.

3. Controller synthesis

The main task of the control design is to regulate the outlet substrate concentration of the tubular bioreactor; it can be done employing the inlet volumetric flow (axial flow velocity) as control input (Dochain *et al.*, 1992). The infinite dimensional model (Eqs. 1-3), was reduced to a finite dimensional model (Eqs. 4-6) discretising the spatial derivatives. In order to reach the control objectives, from the finite dimensional model obtained, only the equations related with to the outlet reactor conditions were considered for the control design; under the assumption that the inner dynamics is stable. Then, the theory of control of lumped parameter systems is applied to the reduced order model, which is a lumped parameter system. It can be rewritten in standard control notation (Eqs. 7-10).

$$\dot{x}_1 = f_1(x_1, x_3) \tag{7}$$

$$\dot{x}_2 = f_2(x_2) \tag{8}$$

$$\dot{x}_3 = f_3(x_1, x_3) + (\bar{B}(x) + \Delta B)u \tag{9}$$

$$y = h(x) = x_3 \tag{10}$$

Here, the following definitions were used:

- $f(x)$ and ΔB are considered model uncertainties related to the non-linear system.
- $\bar{B}(x)$ is the nominal value of the control-input coefficient.
- $u = x_{2o}$ is the system input.
- $x = [x_1, x_2, x_3]$ are the system states.
- $y = x_{3,z=1}$ is the system output.

Now, it is possible to define a convenient change of variables:

$$\eta(x,u) = f_3(x) + \Delta B u \tag{11}$$

By substitution of Eq. (11) into Eqs. (7-10), a new system is obtained (Eqs. 12-15).

$$\dot{x}_1 = f_1(x_1, x_3) \tag{12}$$

$$\dot{x}_2 = f_2(x_1, x_2) \tag{13}$$

$$\dot{x}_3 = \eta(x,u) + \bar{B}(x)u \tag{14}$$

$$y = h(x) = x_3 \tag{15}$$

In order to control this system, let us define the following nominal input-output linearising feedback control (Eq.16).

$$u = \bar{B}^{-1}(x)\left[\tau_g e - \eta(x,u)\right] \tag{16}$$

Here y_{sp} and $e = y - y_{sp}$ are the set point and the regulation error, respectively. The controller defined by Eq. 16 guarantees asymptotic stability of non-linear systems with no uncertainties and perfect measurements (Slotine and Li, 1991), *i.e.* $\Delta B = 0$ and $\eta(x)$ known. It imposes a linear behaviour to the system cancelling the non-linearities. However, since the uncertainty term, $\eta(x,u)$, is unknown and is function of the states, x, and the control input, u, this ideal control law is not causal and therefore is not realisable.

Nonetheless, there is another way to develop an input-output linearising controller that is robust against uncertainties. The procedure shown below defines a method to estimate the uncertainty term, $\eta(x,u)$. This approach is based on observer theory, where the uncertainty is only a function of the estimation error, ε. Let us define the following dynamic subsystem, Eqs. 17-19.

$$\dot{x}_3 = \eta + \bar{B}u \tag{17}$$

$$\dot{\eta} = \Phi(x,u) \tag{18}$$

$$y = \mathrm{h}(x) = x_3 \tag{19}$$

The uncertain term, η, is considered as a new state and $\Phi(x,u)$ is a non-linear unknown function that describes η's dynamics. Firstly, an algebraic observability condition to prove that the pair $\{\eta, x_3\}$ is observable should be performed. From differential algebraic approach (Martínez-Guerra and León-Morales, 1996), Eq., 10, defines an algebraic differential dynamic system. From this subsystem, the following differential-algebraic Eqs. 20, 21, can be obtained:

$$x_3 - y = 0 \tag{20}$$

$$\dot{y} - \bar{B}\,u - \eta = 0 \tag{21}$$

In order to define an algebraically observable uncertainty condition, the following proposition is considered:

Definition 1. An element X_i in a vector field $\aleph$ is said to be an algebraically observable uncertainty if X_i satisfies a differential algebraic equation with coefficients over the field $k\langle u, y\rangle$.

From *Definition 1* and the relationships (20 and 21), it is possible to conclude that the pair $\{\eta, x_3\}$ is universally observable in the Diop-Fliess sense. This means that the uncertain term can be obtained from the input and output of the system and a set of their time derivatives.

From the system given by Eqs. 17-19, it can be seen that a standard observer structure design, *i.e*. a system copy plus output feedback, is not possible since the term Φ is unknown. Considering that variable state X_3 is the system output, lets propose the following uncertainty observer, Eq. 22.

$$\dot{\hat{\eta}} = \tau_1(\eta - \hat{\eta}) + \tau_2(\dot{\eta} - \dot{\hat{\eta}}) \tag{22}$$

This uncertainty estimator is a reduced order observer, which infers the uncertain term (reaction rate) from substrate concentration measurements; η is obtained from the substrate mass balance. Note that the observer contains proportional and derivative actions, the aim of the derivative term is to improve the speed of the estimation algorithm, because it enhances the anticipatory and stabilising effects of derivative actions. This term is necessary because distribute parameter systems lead to time delays in estimation and control actions.

Substituting the estimate of the uncertain term in the ideal controller defined by Eq. 16, the following non-ideal controller is obtained, Eq. 23.

$$u = \bar{B}^{-1}\left[\tau_g e - \hat{\eta}\right] \tag{23}$$

Now, substituting Eq. 23 into Eqs. 17-19, the dynamics of the uncertainty is obtained as:

$$\dot{\eta} = \ddot{e} - \tau_g e + \dot{\hat{\eta}} \tag{24}$$

Introducing this result in Eq. 22 yields:

$$\dot{\hat{\eta}} = \tau_2\,\ddot{e} + (\tau_1 - \tau_2\tau_g)\dot{e} - \tau_g\tau_1 e \tag{25}$$

Since this controller uses an estimated value of the uncertainty, it cannot cancel the system non-linearities, completely. Practical stability is achieved as long as the uncertainty estimation error is bounded (Aguilar *et al.*, 2001). Thus, the system trajectories remain inside a neighbourhood close to the defined set point.

The final expression for the input-output linearising controller with uncertainty estimation can be obtained integrating the estimator (Eq. 25) and substituting it into the non-ideal controller (Eq. 23), to obtain:

$$u = \overline{B}^{-1}(x)\begin{bmatrix}(\tau_g - (\tau_1 - \tau_2\tau_g))e + \\ \tau_g\tau_1\int_o^t e(\theta)d\theta - \tau_2 \dot{e}\end{bmatrix} \quad (26)$$

Note that this controller, Eq. 26, exhibits PID structure and is equivalent to the linearising controller based on proportional-derivative uncertainty observer, Eqs. 22 and 23.

Furthermore, for the resulting PID controller, the tuning procedure is simplified to select the controller gains in a straightforward manner. The controller gains are given in terms of the system structure $\overline{B}(x)$, τ_g, τ_1 and τ_2, which have strong physical meaning for plant operators. The characteristic closed-loop time (τ_g) can be considered as the inverse of the prescribed mean settling time. It is selected same order of the dominant time constant corresponding to the open-loop dynamics of the bioreactor. Hence τ_1 can be chosen same order of the substrate concentration sampling time; τ_2 will be considered later. Therefore, empirical tuning rules for tuning both the controller and the estimator are avoided. This parameterisation is given by Eqs. 27 to 29.

$$K_p = \overline{B}^{-1}(x)\left[\tau_g - (\tau_1 - \tau_2\tau_g)\right] \quad (27)$$

$$\tau_I = \overline{B}^{-1}(x)\tau_1\tau_g \quad (28)$$

$$\tau_D = \overline{B}^{-1}(x)\tau_2 \quad (29)$$

Here K_p is the proportional gain, τ_I is the integral time and τ_D is the derivative time. It is important to note that the controller proposed perform like a PID with time varying gains, because the gains contain the nominal term of the controller coefficient $\overline{B}(x)$.

Note that the aim of the mathematical development presented for the input-output linearising controller based on proportional-derivative reduced order uncertainty observer, is to lead a robust PID control structure, which is implemented for regulation purposes. It is more tractable, generally speaking, for the plant operators and would be easily implemented in commercial hardware (*i.e.* Programming logic controllers).

4. Stability issues

The stability analysis for distributed parameters systems is a difficult task, which is nowadays under develop, despite of some important advances about this topic (Tervo and Nihtila, 2000; Winkin *et al.*, 2000). In order to show the stability properties of the closed-loop system, firstly, a convergence analysis of the uncertainty observer has to be done.

Proposition. Let us define η as the uncertain term and $\hat{\eta}$ as its estimate. The dynamic system $\dot{\hat{\eta}} = \tau_1(\eta - \hat{\eta}) + \tau_2(\dot{\eta} - \dot{\hat{\eta}})$ is an asymptotic-type reduced order observer for the system (17-19).

Proof. Lets to define the estimation error:

$$\varepsilon = \eta - \hat{\eta} \quad (30)$$

Now, the dynamic scalar equation of the estimation error is given by Eq. 31, according to Eqs. 17-19 and 22 is as follows.

$$\dot{\varepsilon} = -\frac{\tau_1}{1+\tau_2}\varepsilon + \frac{\Phi(x,u)}{1+\tau_2} \tag{31}$$

Integrating this last expression, it renders:

$$\begin{aligned}\varepsilon = \varepsilon_0 \, exp\left(-\frac{\tau_1}{1+\tau_2}t\right) + \\ \int_0^t exp\left(\frac{\tau_1}{1+\tau_2}(t-s)\right)\frac{\Phi(x,u)}{1+\tau_2}ds\end{aligned} \tag{32}$$

Now, consider the following assumption: *A1* - $\Phi(x,u)$ is bounded, $\|\Phi\| \le \Psi$, *i.e.* the reaction rate is finite.

Considering the norms of both sides of Eq. 32:

$$\begin{aligned}\|\varepsilon\| \le \|\varepsilon_0\| exp\left(-\frac{\tau_1}{1+\tau_2}t\right) + \\ \int_0^t exp\left(\frac{\tau_1}{1+\tau_2}(t-s)\right)\frac{\|\Phi(x,u)\|}{1+\tau_2}ds\end{aligned}$$

Applying the assumption *A1* the following expression is obtained:

$$\|\varepsilon\| \le exp\left(-\frac{\tau_1}{1+\tau_2}t\right)\left(\|\varepsilon_0\| - \frac{\Psi}{\tau_1}\right) + \frac{\Psi}{\tau_1} \tag{33}$$

In the limit, when $t \to \infty$:

$$\|\varepsilon\| \le \frac{\Psi}{\tau_1} \tag{34}$$

It is important to analyse the structure of the Eq. 33 in order to obtain some characteristics of the proposed observer. As be it is desired, in the limit when $t \to \infty$, the estimation error remain around a closed-ball with radius proportional to $\frac{\Psi}{\tau_1}$, which can be made as small as desired by taken τ_1 large enough.

In order to improve the speed of convergence to the of uncertainty observer convergence to the steady-state estimation error, two actions are have to be taken. The first one is to consider the parameter τ_1 large enough, which is necessary to obtain a small steady-state error, as it was mentioned above. The second one is faced with the influence of the parameter τ_2 related to the derivative action of the uncertainty observer. If $\tau_2 \to -1$, the exponential term of the right side of the Ec. 33 can be accelerated enough and consequently, the convergence of the uncertainty observer will exhibit better performance.

Note that if the measurements of the system are corrupted by additive noise, *i.e.* $y = x_3 + \xi$, and this noise is considered bounded, $\|\xi\| \le \Omega$, a methodology similar to the one used to analyse the estimation error ε can be applied in order to prove that the steady state estimation error becomes $\|\varepsilon\| \le \frac{\Psi+\Omega}{\tau_1}$. This observation confirms robustness against noisy measurements.

Now, it is possible to implement a non-ideal controller using the uncertainty estimated, to produce a practical control law, which can lead the trajectories of substrate concentration in the bioreactor inside a neighbourhood close to the desired set point.

In order to prove closed-loop stability of the outlet substrate concentration in the bioreactor, it is necessary to analyse the closed-loop equation of the substrate mass balance when the non-ideal control law is introduced. The closed-loop balance is shown in Eq. 35, considering the regulation case:

$$\dot{x}_3 = \dot{e} = -\tau_g \, e + (\eta - \hat{\eta}) \tag{35}$$

If $\hat{\eta} \to \eta$, the difference of the right hand side of Eq. 35 is close to zero, then stability properties of this control law can be recovered, as expected, to be analogous to the ideal control. If the difference $(\hat{\eta} - \eta)$ remains bounded, then the estimation error is $\|\varepsilon\| \le \frac{\Psi + \Omega}{\tau_1}$. Now it is possible to solve the linear ordinary differential Eq. (35) for the second statement, and the following inequation is obtained:

$$|e| \le \frac{\Psi + \Omega}{\tau_1 \tau_g} + |e_0| \exp(-\tau_g t) \quad (36)$$

It is easy to note that the limit when $t \to \infty$ is:

$$|e| \le \frac{\Psi + \Omega}{\tau_1 \tau_g} \quad (37)$$

Note that the tracking error e can be made as small as be desired in two ways. The first one is to consider the inverse of the mean settling time τ_g large enough, however this approach has several problems such as great control effort, input saturation and possible instability. The second way consist of good converge of the uncertainty estimator, which needs ε be small enough; this situation can be reached with a high gain of the estimator proposed rather than a high gain of the linearising control.

5. Results and discussion

The open-loop dynamic behaviour of the fixed bed bioreactor is shown in Fig. 1; the numerical values used for parameters can be found in Dochain *et al*. (1992). Several practical situations are introduced as system disturbances in order to prove robustness of the controller proposed. A sustained oscillating sinusoidal perturbation on the substrate input concentration was imposed, it has an amplitude of 5% of the nominal value; besides additive noisy (white noise) around $\pm 3\%$ of the current value of the substrate concentration measurements is considered. The bioreactor was allowed to reach the corresponding open-loop steady state. At $t = 40$ hours the controller was turned-on.

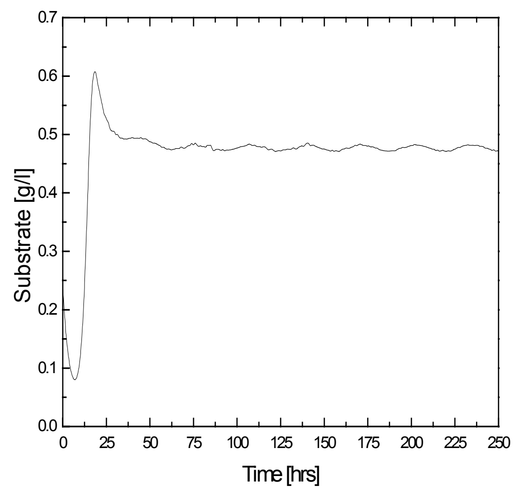

Fig. 1. Open-loop behaviour of the outlet substrate concentration.

In order to compare the performance of standard PID controllers and the proposed controller, numerical simulations were carried out. The standard PID controller was tuned by means of a classical identification methodology based on a step perturbation. The resulting steady-state gain of the process is $K = 4.95\,\mathrm{g \cdot s \cdot mL^{-1}}$, the open-loop characteristic time $\tau = 6.75\,\mathrm{hr}$ and time delay $\alpha = 2.5\,\mathrm{hr}$; IMC and Ziegler-Nichols methods were employed (Ogunnaike and Ray, 1994).

Figs. 2 and 3 show the closed-loop dynamic behaviour when the Ziegler-Nichols tuning method was employed. As it can be seen the performance of the controller is very poor, the system reach periodical oscillations, which are very far of the required set-point ($= 0.26\,\mathrm{g \cdot L^{-1}}$) and the control action effort is too vhigh.

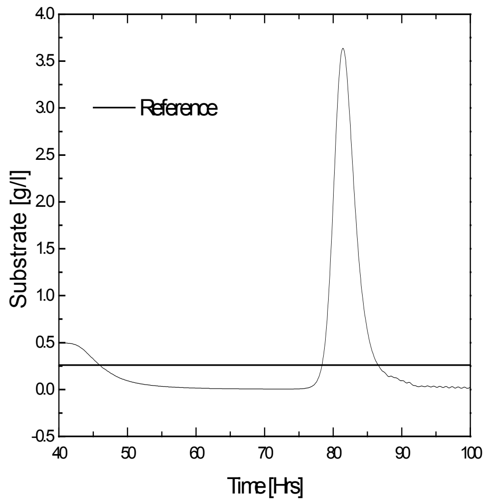

Fig. 2. Closed-loop behaviour of the outlet substrate concentration considering Ziegler-Nichols tuning.

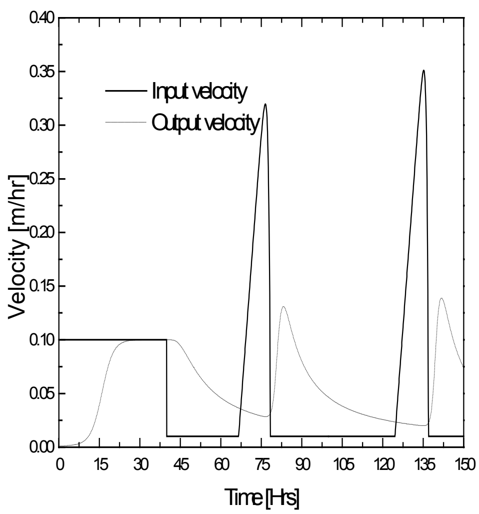

Fig. 3. Closed-loop behaviour of the control input considering Ziegler-Nichols tuning.

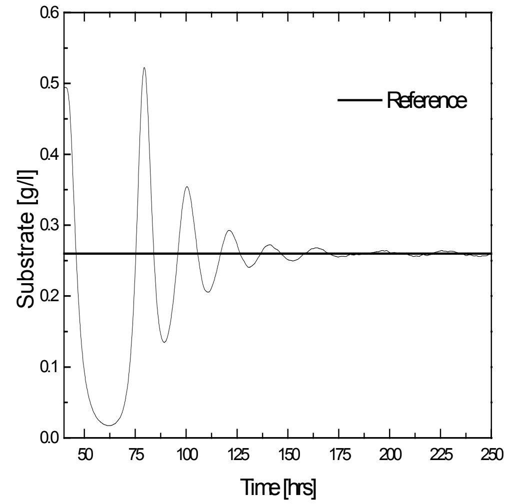

Fig. 4. Closed-loop behaviour of the outlet substrate concentration considering IMC tuning.

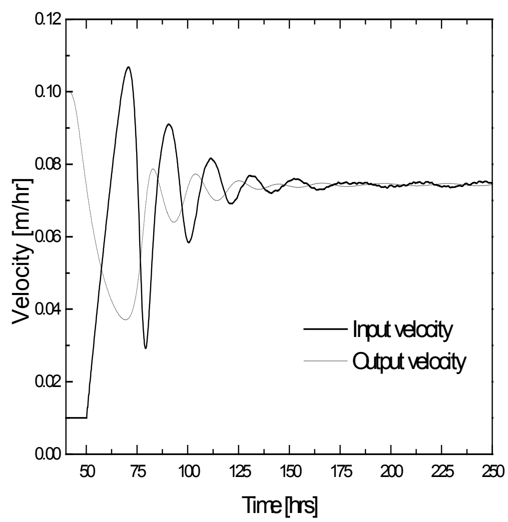

Fig. 5. Closed-loop behaviour of the control input considering IMC tuning.

Subsequently, the IMC tuning rules were employed. Figs. 4 and 5 show the closed-loop performance of the control output and the control input, respectively. It can be observed large settling time in the substrate concentration and overshoot, but despite of this situation the controller can regulate the fixed bed bioreactor, however the performance of the controller cannot be considered satisfactory.

As it can be seen in Figs. 6 and 7, the performance of the controller proposed is very good. The outlet substrate concentration remains in a close neighbourhood around the set-point, according to theoretical developments in previous sections. The settling time and the overshoot are very small compared to the other two tuning methodologies and then the performance of the proposed controller can be considered satisfactory.

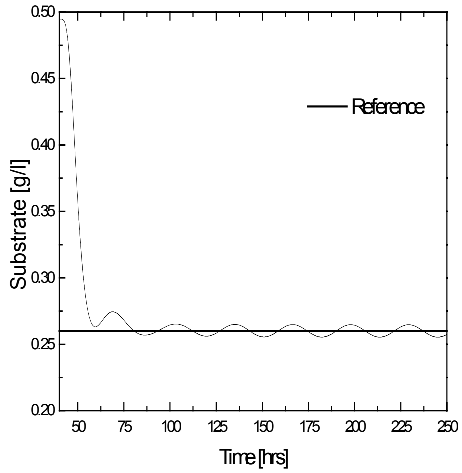

Fig. 6. Closed-loop behaviour of the outlet substrate concentration considering the tuning proposed.

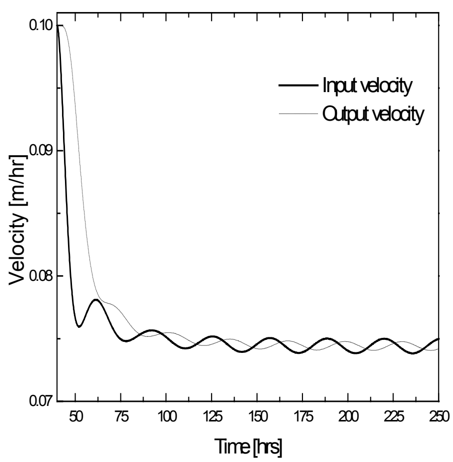

Fig. 7. Closed-loop behaviour of the control input considering the tuning proposed.

Given the time varying structure of the gains of the controller proposed, good robustness and stability against disturbances, modelling errors and noisy measurements were exhibited. Therefore, typical PIDs with fixed gains, simply, could not deal with the disturbances supplied and the highly nonlinear responses of the bioreactor. Finally, Fig. 8 shows the performance of the closed loop behaviour of the uncertainty estimator; it can be observed an adequate convergence of the kinetic term estimated with its *real* evolution.

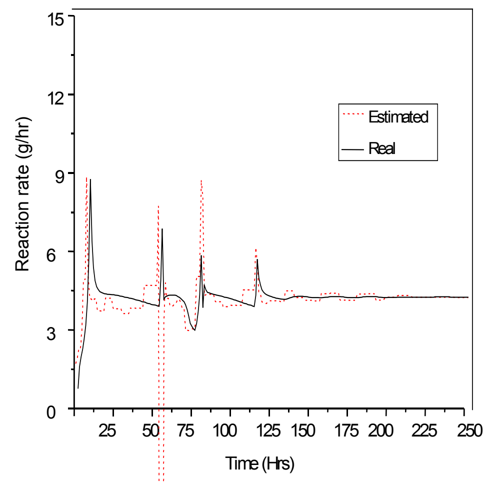

Fig. 8. Uncertainty estimator performance.

Conclusions

A proportional-derivative reduced order observer has been developed in order to produce on-line estimate values of the uncertain term present in a distributed parameters system, in this case related to the reaction rate in a class of fixed bed bioreactor. This observer was coupled to a linearising control law in order to obtain practical closed-loop stability. When the proposed control law is analysed, it is important to observe that it presents the structure of a PID controller with time varying gains. The output substrate concentration was controlled manipulating the input velocity flow, using the PID structure proposed, observing exhibiting satisfactory performance of the process. New tuning rules for the controller gains were given in terms of the system structure, closed-loop and the sampling characteristic times, which have an important physical meaning. This tuning avoids the application of identification methodologies and others methods which propose fixed controller gains (*e.g.* root locus, Ziegler-Nichols, IMC, *etc*.). This novel approach showed better closed-loop performance than other PID controllers applied to systems of distributed parameters.

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